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Lower Hybrid Instability in Ionospheric Gas–Dust Formations from Rocket Exhaust

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I. Introduction

THE launch of powerful rockets, the maneuvering of spacecraft vehicles, and the special modes of rocket-engine operation inject into the atmosphere large amounts of combustion products.[1] These products can affect the ionosphere by generating strong perturbations, such as the large-scale holes in the ionosphere reported by Ref. 2, can induce luminous chemical reactions and lead to optical phenomena caused by the scattering of sunlight on dispersed components of the gas and dust clouds.[1] The nature of such dispersed components depends on the type of rocket engines (solid or liquid propellant) and can have the form of solid particles (e.g., Al, aluminum oxide) or ice particles, with sizes ranging from nanometers to microns.[3] In the ionosphere, the gas-dust formations resulting from rockets' exhaust can have expansion speeds of the order of a few kilometers/second (depending on the altitude) and can reach sizes on the order of 100 km or more (proportional to the breaking distance as presented by Ref. 3), at altitudes ranging from 100 to 700 km and above.

If the dust becomes charged in the background plasma, instabilities may arise because of the relative streaming of the dust and the background ionospheric plasma. Such instabilities might affect radar backscatter from the clouds, when the wave satisfies the Bragg condition for backscatter, $2\boldsymbol{k}_r = \boldsymbol{k}_w$, where $\boldsymbol{k}_r$ is the radar wave number and $\boldsymbol{k}_w$ is the wave number of the wave.[4]

In this Note, we revisit an instability that Ref. 5 had earlier suggested might play a role in explaining the enhanced radar backscatter from space shuttle exhaust observed with the Arecibo 430-MHz radar as reported by Ref. 6. The instability is a lower-hybrid (LH) instability driven by a beam of charged dust streaming across the geomagnetic field with a speed larger than the ion thermal speed. In this Note, we extend the analysis in Ref. 5 to a wider range of parameters and also take into account the effects of charged particle collisions with the background atmospheric molecules. For the parameters that we consider, we find that the instability can lead to short wavelength unstable waves, on the order of tens of centimeters to a meter. We note that recently D'Angelo[7,8] has considered the excitation of longer-wavelength dust modes in the ionosphere, driven by charged dust moving under the action of gravity.

The paper is organized as follows. Section II gives the dispersion relation for the instability, and Sec. III gives numerical results applied to plasmas with solid larger ($\sim$0.1 μm) or icy smaller ($\sim$5 nm) streaming dust particles at altitudes in the upper E/lower F regions of the ionosphere. Section IV summarizes the results.

II. Dispersion Relation

We consider a plasma composed of electrons, singly charged ions, dust of uniform size, and neutrals embedded in a uniform magnetic field $\boldsymbol{B}_0 = B_0 \boldsymbol{z}$. The dust is assumed to be negatively charged as a result of the collection of plasma currents. The condition of overall charge neutrality is

$$n_e + Z_d n_d = n_i \tag{1}$$

where n_α is the number density of species α, with $\alpha = e, i, d$, and n denoting electrons, ions, dust, and neutrals, respectively, and Z_d is the charge state of the dust. We assume the charged dust streams in the y direction with speed V_0.

We consider waves at the LH frequency range, with $\omega_{\rm cd} \ll \omega_{\rm ci} \ll \omega \ll \omega_{\rm ce}$ (here $\omega_{c\alpha}$ is the gyrofrequency of species α), and assume the electrons are magnetized, while the ions and dust are unmagnetized. For the equilibrium, we assume the electrons and ions are described by Maxwellians and the dust by a drifting Maxwellian. We consider the excitation of electrostatic waves with $E_1 \sim -\nabla\phi(x)$ $\exp[i(k_y y + k_z z - \omega t)]$. We assume the charged particles collide primarily with neutrals at the rates ν_α. The resulting kinetic dispersion relation for a LH hybrid instability driven by a charged dust beam streaming across the magnetic field is given in the Appendix.

For purely perpendicular propagation with $k_z = 0$ (so that the waves are not subject to Landau damping[9]), cold ions with $\omega \gg k v_i$ (but with $k v_i \gg \omega_{\rm ci}$), and cold dust with $|\omega - k V_0| \gg k v_d$, we obtain from Eq. (A1)

$$1 + \frac{1-\Gamma_0(b_e)}{(k\lambda_{\rm De})^2}\left(1 + \frac{i\nu_e}{\omega}\right) - \frac{\omega_{\rm pi}^2}{\omega^2}\left(1 - \frac{i\nu_i}{\omega}\right) - \frac{\omega_{\rm pd}^2}{\Omega_b^2}\left(1 - \frac{i\nu_d}{\Omega_b}\right) \approx 0 \tag{2}$$

Here $\Omega_b = \omega - k V_0$; $\omega_{p\alpha}$ and $\lambda_{D\alpha}$ are the plasma frequency and Debye length, respectively, of species α; ρ_e is the electron gyroradius; and $\Gamma_0(b) = I_0(b)\exp(-b)$, where I_0 is the modified Bessel function of zero order. To obtain Eq. (2), we have also assumed that the electron and ion collision frequencies ν_e, ν_i are much smaller than ω and that the dust collision frequency satisfies the condition $\nu_d \ll \Omega_b$.

When collisions can be neglected (this condition will be given next), Eq. (2) becomes

$$1 - \omega_{\rm LH}^2\big/\omega^2 - \omega_{\rm pd}^2\big/P(\omega - kV_0)^2 \approx 0 \tag{3}$$

where the ion lower hybrid type frequency (modified by finite b_e) is given by $\omega_{\rm LH} = \omega_{\rm pi}/\sqrt{P}$, with

$$P = 1 + [1 - \Gamma_0(b_e)]\big/(k\lambda_{\rm De})^2 = 1 + Q$$

Note that $Q \approx \omega_{\rm pe}^2/\omega_{\rm ce}^2$ when $b_e \ll 1$.

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Equation (3) has the form of a dispersion relation for a beam-plasma instability; here, the charged dust comprises the beam, which excites ion lower-hybrid-type waves. Accordingly we take $\omega \approx kV_0 + i\gamma$, with $\gamma \ll kV_0$. The maximum growth rate can be obtained by solving Eq. (3) at the beam-plasma resonance condition $kV_0 \approx \omega_{\rm LH}$, which gives

$$\gamma/\omega_{\rm LH} \approx \left(\sqrt{3}/2\right)(\omega_{\rm pd}/2\omega_{\rm pi})^{\frac{2}{3}} \tag{4}$$

The two-stream condition for instability is given roughly by $kV_0 \leq \omega_{\rm LH}$.

As can be seen from Eqs. (2) and (4), the neglect of electron (and ion) collisional effects on the instability would be justified as long as roughly $(\omega_{\rm pd}/\omega_{\rm pi})^{2/3} \gg \nu_e/\omega$. When this condition is not satisfied, the instability becomes a dissipative-type beam instability. To obtain the maximum growth rate in this case, we retain electron and ion collisions in Eq. (2) and again take $\omega \approx kV_0 + i\gamma \approx \omega_{\rm LH} + i\gamma$, with $\gamma \ll \omega_{\rm LH}$. We then obtain the maximum growth rate in this case as

$$\gamma \approx \left(\omega_{\rm pd}/\sqrt{2}\right)[(Q/P)(\nu_e/\omega_{\rm LH}) + \nu_i/\omega_{\rm LH}]^{-\frac{1}{2}} \tag{5}$$

We note that we have neglected dust collisions because $\nu_d < \omega_{\rm pd}$, whereas maximum growth rates are larger than $\omega_{\rm pd}$.

III. Numerical Results and Applications

In this section we show numerical results obtained by solving the full kinetic dispersion relation, Eq. (A1), applied to a plasma at 1) altitude ~240 km, containing larger 0.1-μm streaming dust grains, and 2) altitude ~160 km, with small ~5-nm-sized streaming ice particulates.

1) As a first case, we consider the altitude region above ~200 km, where Ref. 3 has reported the observation of large-scale dynamic phenomena associated with the shutoff of solid-propellant rocket engines. We consider an altitude $h \sim 240$ km and assume $B \sim 1/2$ G, $n_n \sim 4 \times 10^9$ cm^{-3}, average molecular weight $\mu \sim 20$, $n_i \sim 10^5$ cm^{-3}, and $T_e \sim 2000$ K $\sim 2T_i$. The electron-neutral, ion-neutral collision frequencies are given by[4] $\nu_e \sim 5.4 \times 10^{-10} n_n T_e(K)^{1/2}$ s^{-1} and $\nu_i \sim 2.6 \times 10^{-9}\, n_n/\sqrt{\mu}$ s^{-1}, respectively, where n_n is the neutral density in cm^{-3}. The corresponding ion plasma frequency is $\omega_{\rm pi} \sim 1 \times 10^5$ rad/s, and the ion Debye length is $\lambda_{\rm Di} \sim 0.7$ cm. Dimensionless parameters for the plasma are $\omega_{\rm pi}/\omega_{\rm ci} \sim 370$, $\mu = 20$, $\nu_i/\omega_{\rm pi} = 2 \times 10^{-5}$, $\nu_e/\omega_{\rm pi} = 8 \times 10^{-4}$, and $T_e/T_i \sim 2$.

For the dust, we assume spherical grains of radius $a \sim 0.1$ μm that have mass density ~2 g/cm^3, which yields a dust mass $m_d \sim 4 \times 10^9$ times the proton mass m_p. The relatively large dimension of dust particles associated with this class of phenomena is related to special modes of operation of solid rocket engines (shutoff phase) that occurs usually at altitude above 150 km. During this process, a large quantity of long-lived, large-sized particles (e.g., Al, Al_2O_3) can be expelled along with the exhaust creating the large-scale dynamic phenomena observed by Ref. 3. If the dust is charged negatively by plasma collection currents, the dust charge state would be $Z_d \sim 50$ (estimated using[10] $eZ_d \sim 4aT_e/e$ and assuming isolated grains). To estimate the dust-neutral collision frequency, we use the hard-sphere collision rate $\nu_d \sim 4a^2 n_n v_n m_n/m_d$, where v_n, m_n are the thermal speed and mass, respectively, of the background neutrals,[11] which yields $\nu_d \sim 10^{-3}$ s^{-1}. We assume the density of negatively charged grains is about $n_d \sim 0.006\, n_i$, so that the fraction of negative charge carried by the grains is about 0.3. We also assume that the dust temperature $T_d \sim T_i(\sim T_n)$ and that the dust is ejected with a speed of about $V_0 \sim 2$ km/s into the background plasma perpendicular to the magnetic field, so that $V_0/v_i \sim 3.5$.

Figure 1 shows the real part of the frequency ω_r and the growth rate γ normalized by $\omega_{\rm pi}$ vs $k\lambda_{\rm Di}$. As can be seen, there can be growth with a growth rate $\gamma \sim 2$–40 s^{-1} for a wavelength range spanning about 15 cm–1 m. During a growth time, an artificial dust cloud expanding at the rate of 2 km/s would expand by less than 1 km.

The charging time for the dust grains can be estimated as the timescale for the grain to reach its equilibrium charge, obtained by balancing the electron and ion currents to the grain. This yields a rough estimate for the charging time as $\tau_{\rm ch} \sim \lambda_{\rm Di}/\omega_{\rm pi} a$, and using the preceding parameters yields $\tau_{\rm ch} \sim 1$ s. During this time, the cloud could expand to a radius of about $R \sim 2$ km from its initial injection point, assuming an expansion speed of 2 km/s. For the dust parameters used in Fig. 1, the mass of dust in a cloud with $R \sim 2$ km would be on the order of $n_d m_d R^3 \sim 40$ kg. This value is consistent with the mass of material that is injected into the atmosphere under special modes of rocket-engine operation, such as shutoff of solid fuel rockets, which can be on the order of hundreds of kilograms or more.[1] If the dust density were larger, the instability might persist for a few seconds as the cloud expands.

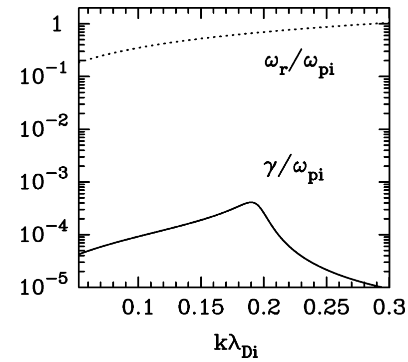

Fig. 1 Real and imaginary parts of the frequency normalized by $\omega_{\rm pi}$ vs $k\lambda_{\rm Di}$, obtained by solving Eq. (A1). Parameters are $\omega_{\rm pi}/\omega_{\rm ci} = 370$, $m_i/m_p = 20$, $T_e = 2T_i = 2T_d$, $k_z = 0$, $\nu_i/\omega_{\rm pi} = 1 \times 10^{-4}$, $\nu_e/\omega_{\rm pi} = 4 \times 10^{-3}$, $Z_d = 50$, $m_d/m_p = 4 \times 10^9$, $n_d/n_i = 0.006$, $\nu_d/\omega_{\rm pi} = 4 \times 10^{-8}$, and $V_0/v_i = 3.5$.

2) Here we consider a plasma at lower altitude ~160 km with streaming nanometer size icy grains. Ice particles can condense in rocket exhaust by the process of water vapor condensation as a result of a rapid expansion of the combustion products.[12] Optical phenomena associated with gas formations containing ice particles have been observed at high altitudes.[3,13] Reference 3 suggests that similar optical phenomena might develop also at lower altitudes ~150 km, but could be hidden by the luminosity of twilight or other brighter optical formations. We suggest that if a dusty plasma instability can occur, perhaps radar backscattering from the unstable waves might be a diagnostic of such artificial ice particles clouds at lower altitudes.

For the plasma, we assume $B \sim 1/2$ G, $n_n \sim 3 \times 10^{10}$ cm^{-3} an average molecular weight $\mu \sim 23$, $n_i \sim 5 \times 10^3$ cm^{-3}, and $T_e \sim 800$ K $\sim T_i$. The corresponding ion plasma frequency is $\omega_{\rm pi} \sim 2 \times 10^4$ rad/s, and the ion Debye length is $\lambda_{\rm Di} \sim 2.5$ cm. Dimensionless parameters for this plasma are $\omega_{\rm pi}/\omega_{\rm ci} \sim 80$, $\mu = 23$, $\nu_i/\omega_{\rm pi} = 1 \times 10^{-3}$, $\nu_e/\omega_{\rm pi} = 3 \times 10^{-2}$, and $T_e/T_i \sim 1$.

For the ice particles, we assume spherical grains of radius $a \sim 5$ nm that have mass density ~1 g/cm^3, which yields a dust mass $m_d \sim 2 \times 10^5$ times the proton mass m_p. Assuming the ice grain is charged negatively by plasma currents, the charge state would be about $Z_d \sim 1$. The dust-neutral collision frequency is $\nu_d \sim 1/2$ s^{-1}, and we assume the density of negatively charged grains is about $n_d \sim 0.6\, n_i$, so that the fraction of negative charge carried by the ice particles is 0.6. We again assume that the ice grain kinetic temperature $T_d \sim T_i(\sim T_n)$ and that the dust flows with a speed of about $V_0 \sim 2$ km/s perpendicular to the magnetic field, so that $V_0/v_i \sim 3.5$.

Figure 2 shows the real part of the frequency ω_r and the growth rate γ normalized by $\omega_{\rm pi}$ vs $k\lambda_{\rm Di}$. The magnitude of the growth rate in this case, $\gamma \sim 2$–200 s^{-1}, is comparable to or larger than that in case 1 just considered. However in this case the unstable waves have longer wavelengths, ranging from about 30 cm to 1 m (which might lead to Bragg backscattering of waves with frequency ~150–400 MHz).

For these small grains that are singly charged, the charging time can be estimated as the time for a grain to collect a single electron. This yields roughly $\tau_{\rm ch} \sim (\pi a^2 n_e v_e)^{-1}$. Using the preceding parameters yields $\tau_{\rm ch} \sim 10$ s. If one assumes that the cloud expansion speed is 2 km/s, the cloud would expand to a radius of about $R \sim 20$ km

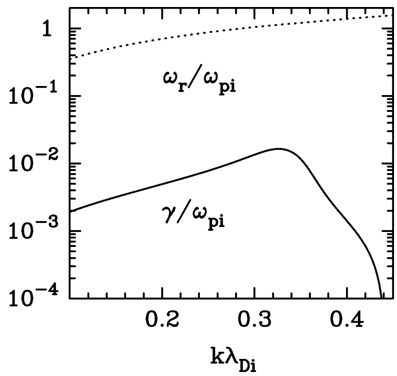

Fig. 2 Real and imaginary parts of the frequency normalized by ω_{pi} vs $k\lambda_{Di}$, obtained by solving Eq. (A1). Parameters are $\omega_{pi}/\omega_{ci} = 80$, $m_i/m_p = 23$, $T_e = T_i = T_d$, $k_z = 0$, $\nu_i/\omega_{pi} = 1 \times 10^{-3}$, $\nu_e/\omega_{pi} = 3 \times 10^{-2}$, $Z_d = 1$, $m_d/m_p = 2 \times 10^5$, $n_d/n_i = 0.6$, $\nu_d/\omega_{pi} = 2 \times 10^{-5}$, and $V_0/v_i = 3.5$.

during the time that the grains get charged. For the parameters of Fig. 2, the mass of the dust in a cloud of radius $R \sim 20$ km would be on the order of $n_d m_d R^3 \sim 8$ kg.

IV. Summary

In this Note we investigated the conditions for a LH instability driven by charged dust particulates (associated with rocket exhaust), streaming across the geomagnetic field in the upper atmosphere. We considered two different sets of possible dusty plasma parameters in the upper E/lower F region (at altitudes ~160 and ~240 km). These involved situations where the dust, which moved across B with speed ~2 km/s, carried a significant fraction of the negative charge density. For the parameters we considered, we found unstable wavelengths ranging from tens of centimeters to on the order of a meter, with instability growth times a fraction of a second. The presence of such unstable waves may create plasma density oscillations that could reflect radar signals, provided that the unstable wave satisfies the Bragg condition for backscatter, namely, $2\boldsymbol{k}_r = \boldsymbol{k}_w$. If the stream speed of the dust were larger (smaller), we would expect the unstable spectrum to shift to somewhat longer (shorter) wavelengths, according to the condition for two-stream instability. We suggest that radar backscatter from such unstable waves, which propagate perpendicular to the magnetic field, might have application as a diagnostic for lower-altitude ice/dust–gas formations arising from rocket exhaust.[3,13]

We note, however, that we have used sets of nominal background plasma parameters and possible dust parameters. Further work should include more detailed evaluations involving ranges of gas, dust, and plasma parameters that might occur in such rocket exhaust clouds at different altitudes.

Appendix: Dispersion Relation

The dispersion relation for a lower-hybrid instability driven by charged dust grains streaming across a magnetic field $B_0\boldsymbol{z}$ with velocity $V_0\boldsymbol{y}$ has been given in Ref. 5. Including collisions, the dispersion relation becomes

$$1 + \chi_e + \chi_i + \chi_d = 0 \tag{A1}$$

where

$$\chi_e = \left[1\big/(k\lambda_{De})^2\right][1 + \zeta_e Z(\zeta_e)\Gamma_0(b_e)] \times \left[1 + \left(i\nu_e\big/\sqrt{2}k_z v_e\right)\Gamma_0(b_e) Z(\zeta_e)\right]^{-1} \tag{A1a}$$

$$\chi_i = \left[1\big/(k\lambda_{Di})^2\right][1 + \zeta_i Z(\zeta_i)]\left[1 + \left(i\nu_i\big/\sqrt{2}kv_i\right)Z(\zeta_i)\right]^{-1} \tag{A1b}$$

$$\chi_d = \left[1\big/(k\lambda_{Dd})^2\right][1 + \zeta_d Z(\zeta_d)]\left[1 + \left(i\nu_d\big/\sqrt{2}kv_d\right)Z(\zeta_d)\right]^{-1} \tag{A1c}$$

Here, $\lambda_{D\alpha} = (4\pi n_\alpha Z_\alpha^2 e^2/T_\alpha)^{-1/2}$ is the Debye length, $v_\alpha = (T_\alpha/m_\alpha)^{1/2}$ is the thermal speed of species α, ν_α is the collision frequency of species α, $b_e = (k_y\rho_e)^2$ with ρ_e being the electron gyroradius, $\Gamma_0(x) = I_0(x)\exp(-x^2)$, where I_0 is the modified Bessel function of zero order, and Z is the plasma dispersion function[14] with the arguments

$$\zeta_e = \frac{\omega + i\nu_e}{\sqrt{2}k_z v_e}$$

$$\zeta_i = \frac{\omega + i\nu_i}{\sqrt{2}kv_i}$$

$$\zeta_d = \frac{\omega - k_y V_0 + i\nu_d}{\sqrt{2}kv_d}$$

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